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$\mathcal{N}=2$ Supersymmetric QCD and Integrable Spin Chains: Rational Case $\mathbf{N}_f < \mathbf{2N}_c$

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Abstract

The form of the spectral curve for $4d$ $\mathcal{N}=2$ supersymmetric Yang-Mills theory with matter fields in the fundamental representation of the gauge group suggests that its $1d$ integrable counterpart should be looked for among (inhomogeneous) $sl(2)$ spin chains with the length of the chain being equal to the number of colours N_c. For $N_f < 2N_c$ the relevant spin chain is the simplest XXX-model, and this identification is in agreement with the known results in Seiberg-Witten theory.

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1 Introduction

Exact form of the abelian low-energy effective actions and BPS massive spectra for the $4d$ $\mathcal{N}=2$ SUSY Yang-Mills (SYM) theories [1]-[5] possess a concise description in terms of $1d$ integrable systems [6]. The *reasons* for this identity remain somewhat obscure, but *the fact* itself is already well established [7]-[11]. To be precise, the relation between the Seiberg-Witten solutions (SW) and integrable theories was so far described in full detail only for particular family of models: the $\mathcal{N}=2$ SYM theory with one ($N_a=1$) "matter" $\mathcal{N}=2$ hypermultiplet in the adjoint representation of the gauge group G – which is known to be related to the Calogero-Moser family of integrable systems [8, 9]. When the hypermultiplet decouples (its mass becomes infinite), the dimensional transmutation takes place and the pure gauge $4d$ $\mathcal{N}=2$ SYM theory gets associated with the Toda-chain model. A physically more interesting family of models – the $\mathcal{N}=2$ supersymmetric QCD (SQCD) with N_f matter $\mathcal{N}=2$ hypermultiplets in *the fundamental* representation of G – does not have a well established integrable counterpart yet. It is known only [10],[11] that the $N_c=3$, $N_f=2$ curve can be associated with the Goryachev-Chaplygin top. The purpose of this letter is to fill, at least partly, this gap. Our suggestion is to associate the family of $\mathcal{N}=2$ SQCD models with the well-known family of integrable systems – (inhomogeneous $sl(2)$) spin chains, of which the Toda chain (pure gauge model) is again a limiting case. The crucial motivation for *such* a suggestion [10] is the peculiar form of the spectral equations, derived in [2], [4]. In this letter we just describe the idea, illustrating it by the simplest example of the rational XXX spin chain, which is, however, enough for the complete description of the $N_f<2N_c$ case. The detailed arguments and analysis of the most interesting elliptic case of $N_f=2N_c$ are postponed to a separate paper.

The SW problem is described by the following set of data (see [9] for details). Let us assume that the YM theory is *softly* regularized both in the UV and IR regions – this is always possible in the $\mathcal{N}=2$ SUSY framework. In the UV region, the theory is embedded – by addition of appropriate massive matter $\mathcal{N}=2$ hypermultiplets – into an UV-*finite* model. At ultra-high energies, this *non-abelian* theory has vanishing β-function, i.e. is conformally-invariant, and possesses a single coupling constant $\tau=\frac{4\pi i}{e^2}+\frac{\theta}{2\pi}$. At the energies below the masses m_α of the additional matter hypermultiplets, the original $\mathcal{N}=2$ SUSY theory is reproduced, which is thus labeled by the set of data $\{G,\tau,m_\alpha\}$.

In the IR region, the theory can avoid entering the strong-coupling regime, if the scalar components of the gauge supermultiplet develop non-zero vacuum expectation values along the valleys of the superpotential. These v.e.v.'s $\langle\Phi\rangle$ are given by diagonal matrices and can be fully described by the set of "moduli" $h_k=\frac{1}{k}\langle \mathrm{Tr}\Phi^k\rangle$. At energies below this IR "soft cutoff", the theory becomes $\mathcal{N}=2$ SUSY *abelian* model, with the *set* of coupling constants T_{ij}. T_{ij} is actually expressed in terms of "periods" a^i, $a_i^D=\frac{\partial\mathcal{F}}{\partial a^i}$: $T_{ij}=\frac{\partial^2\mathcal{F}}{\partial a^i\partial a^j}=\frac{\partial a_i^D}{\partial a^j}$.

The SW problem can be formally defined as a map

$$G,\tau,m_\alpha,h_i\to T_{ij},\ a^i,\ a_i^D \tag{1}$$

and the solution to this problem has an elegant description in the following terms [1, 2]: one associates with the data G,τ,m_α a family of $2d$ surfaces (complex curves) $\mathcal{C}$ with h_i parameterizing (some) moduli of their complex structures, and a meromorphic 1-form dS on every $\mathcal{C}$. Then $a^i=\oint_{A_i}dS$, $a_i^D=\oint_{B^i}dS$. In terms of integrability theory the curves $\mathcal{C}$ are interpreted [6] as the spectral curves of certain integrable systems, and a^i,

a_i^D are related to the action integrals ($\oint pdq$) of the system. Thus, to describe the solution to the SW problem one should present the explicit map

$$G, \tau, m_\alpha \to (\mathcal{C},\ dS)\,\{h_i\}, \tag{2}$$

and this turns out to be equivalent to selection of particular integrable system. The bare charge τ disappears from the formulas in the asymptotically free region $N_f < 2N_c$, where dynamical transmutation substitutes τ by $\Lambda_{QCD}^{(N_f)} \sim \exp \frac{2\pi i\tau}{2N_c - N_f}$. In what follows we put $\Lambda_{QCD}^{(N_f)} = 1$.

2 From Toda to Spin Chains

Our starting point is that the Toda chain spectral curves, corresponding to the pure gauge ($N_f = 0$) $\mathcal{N} = 2$ SUSY theory [6], can be described in terms of two *different* characteristic equations. The first one,

$$\det_{N_c \times N_c} \left(\mathcal{L}^{\rm TC}(w) - \lambda\right) = 0, \tag{3}$$

with $N_c \times N_c$ matrix $\mathcal{L}^{\rm TC}(w)$ being the Lax operator of the periodic Toda chain, can be obtained from a degeneration of the elliptic Calogero-Moser particle system, and this fact is crucially used in description of models with adjoint matter hypermultiplet.

Equation (3) reads

$$w + \frac{1}{w} = 2P_{N_c}(\lambda), \tag{4}$$

due to the very particular form of the matrix $\mathcal{L}^{\rm TC}(w)$ (not preserved by its Calogero-Moser generalization). Here $P_{N_c}(\lambda)$ is a polynomial of degree N_c, whose coefficients are the Schur polynomials of the Toda chain Hamiltonians $h_k = \sum_{i=1}^{N_c} p_i^k + \ldots$:

$$\begin{gathered} P_{N_c}(\lambda) = \sum_{k=0}^{N_c} \mathcal{S}_{N_c - k}(h)\lambda^{N_c} = \\ = \left(\lambda^{N_c} + h_1\lambda^{N_c-1} + \frac{1}{2}(h_2 - h_1^2)\lambda^{N_c-2} + \ldots\right). \end{gathered} \tag{5}$$

Since (4) is quadratic equation with respect to w, one can rewrite it as another characteristic equation involving only 2×2 matrices

$$\det_{2\times 2}\left(T_{N_c}(\lambda) - w\right) = w^2 - w\ {\rm Tr}\ T_{N_c}(\lambda) + \det T_{N_c}(\lambda) = 0. \tag{6}$$

In the Toda-chain case, the 2×2 matrix $T_{N_c}(\lambda)$ is such that ${\rm Tr}\ T_{N_c}^{\rm TC}(\lambda) = P_{N_c}(\lambda)$ and $\det T_{N_c}^{\rm TC}(\lambda) = 1$. According to [2], [4], the spectral curves for the $\mathcal{N} = 2$ SQCD with any $N_f < 2N_c$ have the same form (6) with

$${\rm Tr}\ T_{N_c}(\lambda) = P_{N_c}(\lambda) + R_{N_c-1}(\lambda), \qquad \det T_{N_c}(\lambda) = Q_{N_f}(\lambda), \tag{7}$$

and $Q_{N_f}(\lambda)$ and $R_{N_c-1}(\lambda)$ are certain *h-independent* polinomials of λ.

To go further, let us remind the origin of representation (6) for the Toda-chain theory. The $N_c \times N_c$ Lax equation $\mathcal{L}_{ij}\psi_j = \lambda\psi_i$ can be rewritten through 2×2 matrices [12]:

$$\begin{gathered} \tilde\psi_{i+1} = L_i^{\rm TC}(\lambda)\tilde\psi_i, \\ \tilde\psi_i = \begin{pmatrix} \psi_i \\ \\ \chi_i \end{pmatrix}, \quad L_i^{\rm TC}(\lambda) = \begin{pmatrix} p_i + \lambda & e^{q_i} \\ \\ -e^{-q_i} & 0 \end{pmatrix}, \end{gathered} \tag{8}$$

i.e. $\chi_{i+1} = -e^{-q_i}\psi_i$. Eq.(6) is expressed through the monodromy matrix,

$$T_{N_c}^{\mathrm{TC}}(\lambda) = \prod_{i=N_c}^{1} L_i^{\mathrm{TC}}(\lambda), \quad hboxthus \quad T_{N_c}(\lambda)\tilde{\psi}_i = \tilde{\psi}_{i+N_c} \tag{9}$$

with $\det_{2\times 2} T_{N_c}^{\mathrm{TC}}(\lambda) = \prod_{i=1}^{N_c} \det_{2\times 2} L_i^{\mathrm{TC}}(\lambda - \lambda_i) = 1$ and $\mathrm{Tr}\ T_{N_c}^{\mathrm{TC}}(\lambda) = P_{N_c}(\lambda)$. Eq.(6) can be understood as a corollary of the boundary condition $\tilde{\psi}_{i+N_c} = w\tilde{\psi}_i$. Substitution of (9) into (6) gives rise to the Toda-chain spectral curve (4). Together with the formula for the 1-form $dS = \lambda\frac{dw}{w}$ this provides the solution to the SW problem for pure gauge ($N_f = 0$) theory.

Thus, we reproduce the spectral curve (4) and the 1-form dS of the periodic Toda-chain system from the different perspective – taking a closed chain (of length $= N_c$) of 2×2 Lax matrices and computing the eigenvalues of the monodromy operator. The two descriptions, (3) and (6), are identically equivalent for the Toda chain, but their *deformations* are very different: the "chain" representation (6), (9) is naturally embedded into the family of XYZ spin chains [12, 13], while the $N_c \times N_c$ Lax operator representation – into that of Calogero-Moser models and generic Hitchin systems [14]. Our suggestion is to associate these two different deformations of the integrable Toda chain system with the two different deformations of the pure $\mathcal{N} = 2$ SYM theory: by addition of massive matter multiplets in the *fundamental* and *adjoint* representations of the gauge group $G = SU(N_c)$ respectively. Self-consistency of the $4d$ theory in the UV region requires that $N_f \le 2N_c$ and $N_a \le 1$, thus the numbers of deformation parameters (masses m_α) in the two cases are $2N_c$ and 1. Since adjoint model is exhaustively analyzed in [8, 9], in what follows we concentrate on the fundamental case.

Integrability of the Toda chain in representation (8) follows from *quadratic* r-matrix relations [13]

$$\left\{L(\lambda) \overset{\otimes}{,} L(\lambda')\right\} = \left[r(\lambda - \lambda'),\ L(\lambda) \otimes L(\lambda')\right], \tag{10}$$

so that $\{p_i, q_j\} = \delta_{ij}$ follows from (10) with the rational r-matrix (see (18) below). The crucial property of this relation is that it is multiplicative and any product like (9) satisfies the same relation

$$\left\{T_{N_c}(\lambda) \overset{\otimes}{,} T_{N_c}(\lambda')\right\} = \left[r(\lambda - \lambda'),\ T_{N_c}(\lambda) \otimes T_{N_c}(\lambda')\right], \tag{11}$$

provided all L_i in product (9) are independent, $\{L_i, L_j\} = 0$ for $i \neq j$.

Our proposal is to look at non-Hitchin generalizations of the Toda chain, i.e. deform eqs.(6)-(9) preserving the quadraticity of Poisson brackets (11) and, thus, the possibility to build a monodromy matrix $T(\lambda)$ by multiplication of $L_i(\lambda)$'s. For a moment, we even allow $L(\lambda)$ to be $n \times n$, not obligatory 2×2 matrices.

The full spectral curve for the periodic *inhomogeneous* spin chain is given by:

$$\det_{n\times n}\left(T_{N_c}(\lambda) - w\right) = 0, \tag{12}$$

with the inhomogeneous T-matrix

$$T_{N_c}(\lambda) = \prod_{i=N_c}^{1} L_i(\lambda - \lambda_i) \tag{13}$$

still satisfying (11), and d^{-1}(symplectic form) is now

$$dS = \lambda\frac{d\tilde{w}}{\tilde{w}}, \qquad \tilde{w} = w \cdot (\det T_{N_c})^{-1/n}. \tag{14}$$

In the particular case of $\mathcal{N}=2$ ($sl(2)$ spin chains), the spectral equation acquires the form (6) (in general the spectral equation is of the n-th order in w):

$$w + \frac{\det_{2\times 2} T_{N_c}(\lambda)}{w} = \mathrm{Tr}_{2\times 2} T_{N_c}(\lambda), \tag{15}$$

or

$$\tilde{w} + \frac{1}{\tilde{w}} = \frac{\mathrm{Tr}_{2\times 2} T_{N_c}(\lambda)}{\sqrt{\det_{2\times 2} T_{N_c}(\lambda)}}. \tag{16}$$

The r.h.s. of this equation contains the dynamical variables of the spin system only in the special combinations – its Hamiltonians (which are all in involution, i.e. Poisson-commuting). It is this peculiar shape (quadratic w-dependence) that suggests the identification of the periodic $sl(2)$ spin chains with solutions to the SW problem with the fundamental matter supermultiplets.

3 XXX Spin Chain and the Low Energy SYM with $N_f < 2N_c$

The 2×2 Lax matrix for the $sl(2)$ XXX chain is

$$L(\lambda) = \lambda \cdot \mathbf{1} + \sum_{a=1}^{3} S_a \cdot \sigma^a. \tag{17}$$

The Poisson brackets of the dynamical variables S_a, $a = 1, 2, 3$ (taking values in the algebra of functions) are implied by (10) with the rational r-matrix

$$r(\lambda) = \frac{1}{\lambda}\sum_{a=1}^{3} \sigma^a \otimes \sigma^a. \tag{18}$$

In the $sl(2)$ case, they are just

$$\{S_a, S_b\} = i\epsilon_{abc} S_c, \tag{19}$$

i.e. $\{S_a\}$ plays the role of angular momentum ("classical spin") giving the name "spin-chains" to the whole class of systems. Algebra (19) has an obvious Casimir operator (an invariant, which Poisson commutes with all the generators S_a),

$$K^2 = \mathbf{S}^2 = \sum_{a=1}^{3} S_a S_a, \tag{20}$$

so that

$$\begin{gathered}
\det_{2\times 2} L(\lambda) = \lambda^2 - K^2, \\
\det_{2\times 2} T_{N_c}(\lambda) = \prod_{i=N_c}^{1} \det_{2\times 2} L_i(\lambda - \lambda_i) = \prod_{i=N_c}^{1} \left((\lambda-\lambda_i)^2 - K_i^2\right) = \\
= \prod_{i=N_c}^{1} (\lambda + m_i^+)(\lambda + m_i^-) = Q_{2N_c}(\lambda),
\end{gathered} \tag{21}$$

where we assumed that the values of spin K can be different at different nodes of the chain, and [1]

$$m_i^{\pm} = -\lambda_i \mp K_i. \tag{22}$$

[1] Eq.(22) implies that the limit of vanishing masses, all $m_i^{\pm} = 0$, is associated with the *homogeneous* chain (all $\lambda_i = 0$) and vanishing spins at each site (all $K_i = 0$). It deserves noting that a similar situation was considered by L.Lipatov [15] in the study of *the high*-energy limit of the ordinary (non-supersymmetric) QCD. The spectral equation is then the classical limit of the Baxter equation from [16].

While the determinant of monodromy matrix (21) depends on dynamical variables only through Casimirs K_i of the Poisson algebra, the dependence of the trace $\mathcal{T}_{N_c}(\lambda) = \frac{1}{2}\mathrm{Tr}_{2\times 2}T_{N_c}(\lambda)$ is less trivial. Still, as usual for integrable systems, it depends on $S_a^{(i)}$ only through Hamiltonians of the spin chain (which are not Casimirs but Poisson-commute with *each other*).

In order to get some impression how the Hamiltonians look like, we present explicit examples of monodromy matrices for $N_c = 2$ and 3. Hamiltonians depend non-trivially on the λ_i-parameters (inhomogeneities of the chain) and the coefficients in the spectral equation (12) depend only on the Hamiltonians and symmetric functions of the m-parameters (22), i.e. the dependence of $\{\lambda_i\}$ and $\{K_i\}$ is rather special. This property is crucial for identification of the m-parameters with the masses of the matter supermultiplets in the $\mathcal{N} = 2$ SQCD.

$\mathbf{N}_c = \mathbf{2}$

$$\begin{gathered}\mathcal{T}_2(\lambda) = (\lambda-\lambda_1)(\lambda-\lambda_2) - \sum_{a=1}^{3} S_a^{(1)}S_a^{(2)} = \\ = \lambda^2 - (\lambda_1+\lambda_2)\lambda + \left(h_2 + \lambda_1\lambda_2 - \frac{1}{2}(K_1^2+K_2^2) - \frac{1}{2}(\lambda_1^2+\lambda_2^2)\right).\end{gathered} \tag{23}$$

The second Hamiltonian is

$$\begin{gathered}h_2\{\lambda_i\} = -\sum_{a=1}^{3}\sum_{i<j}^{N_c} S_a^{(i)}S_a^{(j)} - \frac{1}{4}t_1(K^2) - \frac{1}{4}t_1(\lambda^2) = \\ \overset{N_c=2}{=} -\sum_{a=1}^{3} S_a^{(1)}S_a^{(2)} - \frac{1}{4}(K_1^2+K_2^2) - \frac{1}{4}(\lambda_1^2+\lambda_2^2).\end{gathered} \tag{24}$$

The coefficient of the λ^1-term at the r.h.s. of (23) can be expressed through the parameters $m^{\pm}$, defined by (22):

$$-(\lambda_1+\lambda_2) = \frac{1}{2}(m_1^+ + m_1^- + m_2^+ + m_2^-) = \frac{1}{2}\sum_{\gamma=1}^{2N_c} m_\gamma = \frac{1}{2}t_1\{m\}. \tag{25}$$

where we introduced an obvious notation $\{m_\gamma\}$ for the whole set of parameters $\{m_i^{\pm}\}$, and the symmetric functions are defined as

$$t_k\{m\} = \sum_{\gamma_1<\ldots<\gamma_k} m_{\gamma_1}\ldots m_{\gamma_k} \tag{26}$$

for any sets of variables.

The last (λ^0) term at the r.h.s. of (23) can be represented as

$$\begin{gathered}h_2 + \lambda_1\lambda_2 + \frac{1}{4}(K_1^2+K_2^2) + \frac{1}{4}(\lambda_1^2+\lambda_2^2) = \\ = h_2 + t_2(\lambda) + \frac{1}{4}t_1(K^2) + \frac{1}{4}t_1(\lambda^2) = h_2\{\lambda_i\} + \frac{1}{4}t_2\{m\}.\end{gathered} \tag{27}$$

Indeed,

$$\begin{gathered}t_2\{m\} = \frac{1}{2}\left(\left(\sum_{\gamma=1}^{2N_c} m_\gamma\right)^2 - \sum_{\gamma=1}^{2N_c} m_\gamma^2\right) = \\ = \frac{1}{2}\left(\left(2\sum_{i=1}^{N_c}\lambda_i\right)^2 - 2\sum_{i=1}^{N_c}\left(\lambda_i^2 - K_i^2\right)\right) = 4t_2(\lambda) + t_1(\lambda^2) + t_1(K^2).\end{gathered} \tag{28}$$

$\mathbf{N}_c = \mathbf{3}$

In this case:

$$\begin{gathered}\mathcal{T}_3(\lambda) = \lambda^3 - (\lambda_1 + \lambda_2 + \lambda_3)\lambda^2 + \\ + \left(\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1 - \sum_{a=1}^{3}(S_a^{(1)}S_a^{(2)} + S_a^{(2)}S_a^{(3)} + S_a^{(3)}S_a^{(1)})\right)\lambda + \\ + i\epsilon_{abc}S_a^{(1)}S_b^{(2)}S_c^{(3)} = \\ = \lambda^3 + \frac{1}{2}t_1\{m\}\lambda^2 + (h_2 + \frac{1}{4}t_2\{m\})\lambda + (h_3 + \frac{1}{8}t_3\{m\}),\end{gathered} \tag{29}$$

where $h_2\{\lambda_i\}$ has been already defined in (24) and

$$\begin{gathered}h_3\{\lambda_i\} = i\epsilon_{abc}\sum_{i<j<k}^{N_c} S_a^{(i)}S_b^{(j)}S_c^{(k)} + \sum_i \sum_{\substack{j,k\neq i \\ j<k}} \lambda_i S_a^{(j)}S_a^{(k)} + \\ + \frac{1}{4}\left(\left[t_1(\lambda^2) + t_1(K^2)\right]t_1(\lambda) - t_1(\lambda^3) - t_1(\lambda K^2)\right),\end{gathered} \tag{30}$$

while

$$\begin{gathered}t_3\{m\} = \frac{1}{6}\left(\left(\sum m\right)^3 - 3\left(\sum m^2\right)\left(\sum m\right) + 2\sum m^3\right) = \\ = -8t_3(\lambda) - 2t_1(\lambda^2)t_1(\lambda) - 2t_1(K^2)t_1(\lambda) + 2t_1(\lambda^3) + 2t_1(\lambda K^2).\end{gathered} \tag{31}$$

Similarly one can deduce that

$$\mathcal{T}_{N_c}(\lambda) = \frac{1}{2}\mathrm{Tr}_{2\times 2}T_{N_c}(\lambda) = P_{N_c}(\lambda|h) + \sum \lambda^{N_f - N_c - i}t_i\{m\} = P_{N_c}(\lambda|h) + R_{N_c-1}(\lambda|m). \tag{32}$$

Together with (14)-(16) and (21), this reproduces the formulas proposed in [4].

Thus, we demonstrated that the SW problem for the $\mathcal{N} = 2$ SUSY QCD with $N_f < 2N_c$ is solved in terms of integrable XXX spin chain. This construction has a natural elliptic generalization, which describes the conformal point $N_f = 2N_c$. The details will be presented elsewhere.

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